Are Model Parameters Linked to Processing Stages? An Empirical Investigation for the ex-Gaussian, ex-Wald, and EZ Diffusion Models

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Abstract

In previous research, the parameters of the ex-Gaussian distribution have been subject to a wide variety of interpretations. The present study investigated whether the ex-Gaussian model is capable of distinguishing effects on separate processing stages (i.e., pre-motor vs. motor). In order to do so, we used datasets where the locus of effect was quite clear. Specifically, we analyzed data from experiments comparing hand vs. foot responses — presumably differing in the motor stage — and from experiments in which the lateralized readiness potential was used to localize experimental effects into premotor vs. motor processes. Moreover, we broadened the scope to two other descriptive RT models: the ex-Wald and EZ diffusion models. To the extent possible with each of these models, we reanalyzed the RT data of 19 clearly localized experimental effects from 12 separate experiments reported in seven previously-published articles. Unfortunately, we did not find a clear pattern of results for any of the models, with no clear link between effects on one of the model's parameters and effects on different processing stages. The present results suggest that one should resist the temptation to associate specific processing stages with individual parameters of the ex-Gaussian, ex-Wald, and EZ diffusion models.

Keywords: RT Distributional Analysis, ex-Gaussian, ex-Wald, EZ Diffusion, Hand vs. Foot Responses, Lateralized Readiness Potential

Are Model Parameters Linked to Processing Stages? An Empirical Investigation for the ex-Gaussian, ex-Wald, and EZ Diffusion Models

In the field of cognitive psychology, the most frequently used dependent variable is the reaction time (RT), usually measured in rather simple tasks (e.g., Luce, 1986; Matzke & Wagenmakers, 2009; Van Zandt, 2000; Whelan, 2008). Most commonly, researchers use the means of RTs in an analysis of variance (ANOVA) in order to draw conclusions about the underlying cognitive mechanisms (Van Zandt, 2002; Whelan, 2008). However, as RT distributions are usually not normally distributed but positively skewed (Heathcote, Popiel, & Mewhort, 1991), it has often been argued that by simply using the mean of RTs, important information about the underlying cognitive mechanisms can get lost (Balota & Yap, 2011; Heathcote et al., 1991; Matzke & Wagenmakers, 2009; Van Zandt, 2000; Whelan, 2008). More specifically, the behavior of fast and slow responses can be affected differently by experimental manipulations (Heathcote et al., 1991; Ridderinkhof, Scheres, Oosterlaan, & Sergeant, 2005; Van Zandt, 2000), possibly even affecting the RT distribution without affecting the mean (Spieler, Balota, & Faust, 2000). Heathcote et al. (1991) go so far as to suggest "that RT measures should always be analyzed using a distributional analysis and that the traditional analysis of M_{RT} [mean RT] risks serious misinterpretation of the data" (p. 340).

A number of approaches have been developed for analyzing the distributional properties of RT data (e.g., Brown & Heathcote, 2008; Ratcliff, 1978, 1979; Schwarz, 2001; Van Zandt, 2000; Wagenmakers, Van Der Maas, & Grasman, 2007). Although these approaches differ conceptually, their general goal is to link particular model parameters with particular underlying mental processes in order to draw conclusions about specific cognitive mechanisms. This goal is most certainly attractive, because it could potentially allow for more detailed analyses of these mechanisms than are possible using just the mean RT. To justify this kind of analysis, however, a model must be validated empirically by showing that well-understood experimental manipulations have the predicted effects on particular model parameters (e.g., Voss, Rothermund, & Voss, 2004).

Rationale of the Present Study

This study examined three models capable of describing — at least to some extent — distributional properties of RT. These were the ex-Gaussian model (Burbeck & Luce, 1982; Hohle, 1965), the ex-Wald model (Schwarz, 2001), and the EZ diffusion model (Wagenmakers et al., 2007), which are described in the following sections. To the extent possible with each of these models, we reanalyzed the RT data from 12 separate experiments reported in seven previously-published articles. These 12 experiments were chosen because in each case there were good reasons to believe that at least one experimental manipulation clearly influenced a processing stage specifically either before or after the onset of motor response preparation. Thus, the aim of the present study was to investigate the effects on model parameters of experimental manipulations with a clear locus of effect. In particular, it is important to ask whether model parameters correspond selectively to manipulations affecting solely cognitive or motor processes, because an affirmative answer to that question for any model would make that model an especially attractive tool for the analysis of RT data.

There were two groups of data sets. The first involved comparisons between hand and foot responses. Foot responses are generally much slower than hand responses, and it seems extremely plausible that most if not all of the RT differences between the two effectors come from differences in the motor-related component of the RT. Thus, if one model parameter corresponds uniquely to the hand versus foot manipulation, it would be reasonable to conclude that this parameter represents the motor component of the RT distribution within that model.

The second group of data sets involved experimental manipulations that had been independently localized using the lateralized readiness potential (LRP; see Smulders & Miller, 2012, for an overview). Voluntary hand movements (such as discrete responses to RT tasks) are preceded by a lateralized *Bereitschaftspotential* (Deecke, Grözinger, & Kornhuber, 1976) that can be seen in electroencephalographic (EEG) activity over the sensorimotor cortical areas contralateral versus ipsilateral to the responding hand. This LRP can be isolated from other EEG components and is a reliable measure of hand-specific motor preparation (Osman, Moore, & Ulrich, 1995; Smulders, Kok, Kenemans, & Bashore, 1995; Smulders & Miller, 2012). Thus, the onset of the LRP provides a specific time marker that can be used to divide the total RT between stimulus onset and response into subintervals preceding versus following the onset of motor preparation (Smulders & Miller, 2012). The locus of experimental manipulations can thus be distinguished as being either before the start of motor preparation (i.e., stimulus-locked LRP effects) or after that point (i.e., response-locked LRP effects), and this method of effect localization has been used in many previous studies (e.g., Hackley & Valle-Inclan, 1998; Miller & Ulrich, 1998; Osman & Moore, 1993; Smulders et al., 1995; see Table 1 of Smulders & Miller, 2012, for many additional examples). In the present study, we reanalyzed the RT data from a number of LRP studies to see which RT model parameters were affected by the experimental manipulations. If different model parameters are selectively affected by manipulations that had been found to have effects either before or after LRP onset, the interpretations of these parameters would thus be clarified.

In summary, our approach was similar to that of Voss et al. (2004), who examined the effects of certain well-understood experimental manipulations on the parameters of the Ratcliff (1978) diffusion model (for other validation studies of the Ratcliff diffusion model, see, e.g., Arnold, Bröder, & Bayen, 2014; Dutilh et al., 2018; Gomez, Ratcliff, & Childers, 2015; Lerche & Voss, 2017). We reanalyzed the RT data obtained in previously-published experiments whose experimental manipulations could be relatively clearly identified with particular processing stages, and we checked to see how those manipulations affected the parameters of the ex-Gaussian, ex-Wald, and EZ diffusion models. Our overall goal was to see whether the manipulations associated with particular stages consistently affected certain model parameters, as would be expected if the model parameters correspond to specific stages.

The ex-Gaussian Distribution

One approach for analyzing RT distributions is to describe them as ex-Gaussian distributions. Mathematically, the ex-Gaussian distribution is the sum (i.e., convolution) of independent Gaussian and exponential components (Burbeck & Luce, 1982; Hohle, 1965; Luce, 1986). The Gaussian portion is captured by two parameters, the mean μ and standard deviation σ, and the exponential portion is described by its mean τ. The mean and variance of the overall ex-Gaussian sum are thus

$$\begin{aligned} \mathrm{E}(x) &= \mu + \tau, \text{ and} \\ \mathrm{Var}(x) &= \sigma^2 + \tau^2. \end{aligned}$$

Empirically, the ex-Gaussian distribution often provides a good fit to observed RT distributions (Hohle, 1965; Luce, 1986). Since the ex-Gaussian is defined by its three parameters μ, σ, and τ, these parameters can be estimated for each participant and condition, and the parameter values can then be analyzed as separate descriptors of the overall RT distribution (e.g., using ANOVAs). As described next, this makes it possible to study the separate effects of experimental manipulations on three separate distributional descriptors rather than just on the mean RT.

The concept of estimating and comparing ex-Gaussian parameters is fairly old (e.g., Gholson & Hohle, 1968a, 1968b; Hohle, 1965) and has been used in a wide range of research areas. One such area is research on ADHD (e.g., Buzy, Medoff, & Schweitzer, 2009; Castellanos, Sonuga-Barke, Milham, & Tannock, 2006; Epstein et al., 2011; Gmehlin et al., 2014; Hervey et al., 2006; Izawa et al., 2012; Kóbor et al., 2015; Lee et al., 2015; Leth-Steensen, Elbaz, & Douglas, 2000; Tarantino, Cutini, Mogentale, & Bisiacchi, 2013; Vaurio, Simmonds, &

Mostofsky, 2009; see Tamm et al., 2012 for a review). For instance, Kóbor et al. (2015) used the ex-Gaussian distribution to study the differences between children with ADHD and a control group. Participants performed an animal Stroop task with compatible and incompatible trials. Just looking at the results using mean RT as the dependent variable, they found that children with ADHD responded significantly more slowly than the control group. Moreover, Kóbor et al. found a main effect of compatibility, with faster mean RTs in compatible than in incompatible trials. Additionally, they compared the ex-Gaussian parameters estimated separately for each participant and compatibility condition. The compatibility factor was only significant in the analysis of μ, whereas the group factor (ADHD vs. control) was only significant for τ. Especially the finding of increased τ for the ADHD group is well established (Tamm et al., 2012), and this increase is usually attributed to occasional lapses in attention in the patient group (Kóbor et al., 2015; Leth-Steensen et al., 2000; Tamm et al., 2012). This interpretation seems quite plausible because inattention is one of the central criteria for the diagnosis of ADHD (American Psychiatric Association, 2013).

Similarly, the ex-Gaussian distribution has also been applied within research on Alzheimer's disease (e.g., Balota et al., 2010; Gordon & Carson, 1990; Jackson, Balota, Duchek, & Head, 2012; Spieler, Balota, & Faust, 1996; Tse, Balota, Yap, Duchek, & McCabe, 2010) and on memory retrieval processes (e.g., Rohrer, 1996, 2002; Rohrer & Wixted, 1994; Wixted, Ghadisha, & Vera, 1997; Wixted & Rohrer, 1993). Distributional analyses using the ex-Gaussian model in Alzheimer's research seem to link higher τ values to either currently having a form of Alzheimer's disease (Gordon & Carson, 1990; Jackson et al., 2012; Spieler et al., 1996; Tse et al., 2010) or later developing one (Balota et al., 2010). These greater τ values have been interpreted as reflecting breakdowns in executive control processes for Alzheimer's patients, specifically in attentional processes (Balota et al., 2010; Jackson et al., 2012; Tse et al., 2010). Within memory retrieval research, the recall process is generally assumed to be exponential

(Rohrer, 1996), and the ex-Gaussian shape only develops because of an initial pause preceding the recall (Rohrer, 1996, 2002; Rohrer & Wixted, 1994; Wixted et al., 1997).

Within chronometric studies of the mental processes used in standard choice RT tasks, one area of common interest in studies using the ex-Gaussian distribution is in distinguishing between the more central processes involved in decision making and other, more peripheral processes required by the task (i.e., sensory and motor processes). Since there is no theoretical foundation for the ex-Gaussian distribution, however, the interpretation of the ex-Gaussian parameters in terms of these processes is not straightforward. In fact, from the earliest work with the ex-Gaussian model, there have been stark differences in the assignment of the Gaussian versus exponential components to decision versus sensorimotor processes (see Matzke & Wagenmakers, 2009 for an overview; also see Schwarz, 2001). For example, Christie and Luce (1956) argued that decisional processes are best represented by an exponential distribution, and Hohle (1965) also suggested that τ reflects the decisional component of RTs, whereas μ represents the motor-related component. In contrast, McGill (1963) and McGill and Gibbon (1965) offered exactly the opposite interpretation. Obviously, although the ex-Gaussian distribution can be used as a merely descriptive tool for observed choice RTs (Schwarz, 2001), it would be more useful for RT analysis if its parameters could be identified uniquely with decisional and sensorimotor components — one way or the other.

In recent years, a consensus has begun to emerge linking the μ component to sensorimotor processes and the τ component to more central decision processes. This view is supported by evidence that the effects of response conflict (e.g., in the task of Stroop, 1935) tend to show up mainly on μ, because there is independent evidence that response conflict affects motor processing (e.g., Praamstra & Seiss, 2005; Verleger, Kuniecki, Möller, Fritzmannova, & Siebner, 2009). In contrast, the effects of higher-level manipulations such as task-switching tend to show up in τ (e.g., Moutsopoulou & Waszak, 2012; Steinhauser & Hübner, 2009). Summarizing the state of the literature, Singh, Laub, Burgard, and Frings (2018) concluded that the ex-Gaussian is "effective in analyzing the effects of cognitive and motor processes independently ... [and] τ reflects conflict in higher cognitive processes and μ reflects conflict in motor processes" (p. 798).

As there have been contrasting arguments about whether the motor-related component of RTs is reflected in the Gaussian component (Hohle, 1965; Moutsopoulou & Waszak, 2012; Singh et al., 2018) or the exponential component (McGill, 1963; McGill & Gibbon, 1965), it seems worthwhile to examine in more detail this emerging consensus regarding the μ and τ components of the ex-Gaussian model. In particular, it is important to ask whether these components correspond selectively to manipulations affecting solely cognitive or motor processes, because an affirmative answer to that question would make the ex-Gaussian model an especially attractive tool for the analysis of RT data. Thus, with respect to the ex-Gaussian distribution, the main aim of the present study was to investigate the effects on ex-Gaussian parameters of experimental manipulations with a clear locus of effect.

The ex-Wald Distribution and the EZ Diffusion Model

Although our initial emphasis was on the ex-Gaussian distribution, we broadened the scope of our RT analyses to include two additional models. The second model was the ex-Wald distribution (Schwarz, 2001), which is similar to the ex-Gaussian in that it provides good fits to observed RT distributions and furnishes additional parameters for describing their shapes. Its major advantage over the ex-Gaussian is that it has a stronger theoretical rationale (Schwarz, 2001).

The ex-Wald distribution describes RTs as a convolution of the Wald distribution with an exponential distribution. Thus, the main difference from the ex-Gaussian is that the non-exponential portion follows a Wald distribution instead of a Gaussian distribution. Since the

Wald distribution represents a Wiener diffusion process, it is by definition non-negative (Schwarz, 2001)[1].

In this article, we denote the ex-Wald parameters parallel to the ex-Gaussian parameters for ease of comparison. That is, the mean and standard deviation of the Wald portion of the ex-Wald distribution are denoted by μ and σ, respectively, whereas the exponential portion of the ex-Wald distribution is denoted by τ.

The ex-Wald distribution also offers the possibility of estimating μ, σ, and τ separately for each participant and condition, and it is possible to interpret the ex-Wald parameters in a manner parallel to the parameters of the ex-Gaussian model. However, even though the ex-Wald model is more theoretically sound than the ex-Gaussian model, it is much newer and has thus not been widely employed in this fashion.

The third model that we examined was a more process-oriented model — namely, the EZ diffusion model of Wagenmakers et al. (2007). This model can be used to subdivide the total RT into separate components representing decision-related versus residual processes, somewhat analogous to the two components of the ex-Gaussian and ex-Wald distributions. Wagenmakers et al. (2007) introduced the EZ diffusion model as a simplified, rather descriptive version of the full Ratcliff (1978) diffusion model. In the full diffusion model, a decision process is driven by noisy information accumulation over time. When the accumulated evidence for one of the two choice alternatives reaches a threshold, that response is executed (Ratcliff, 1978). In general, diffusion models have been found to provide good descriptions of RT distributions in many two-alternative forced-choice tasks (e.g., Ratcliff & Smith, 2004; Wagenmakers et al., 2007).

[1] Schwarz (2001) interprets the diffusion process as an accumulation of noisy partial information over time. Thus, the ex-Wald model has one additional noise parameter which is just an arbitrary scaling parameter usually set to one for the parameter estimations. See Schwarz (2001) for further methodological and mathematical details.

However, it is impractical to estimate the seven parameters of the full diffusion model in experiments with high accuracy, because the full RT distribution for erroneous responses is needed, and this distribution cannot be determined accurately when errors are rare.

The EZ diffusion model is a simplified version of the diffusion model with only three parameters: drift rate v (i.e., quality of information), boundary separation a (i.e., response conservativeness), and the non-decision time T_{er} (i.e., time for pre-decision-stage perceptual processes and post-decision motor processes). It can be used in more contexts than the full diffusion model because these parameters can be estimated using only the percentage of correct responses and the mean and variance of the correct RT[2].

To make the analyses of the EZ diffusion model more comparable to those of the ex-Gaussian and ex-Wald models, we used this model to decompose overall mean RTs into components associated with the mean decision time T_D and the mean residual time T_{er}. In our analyses of manipulations with a clear locus of effect, one would expect the decisional component T_D to correspond uniquely to manipulations with a more central locus of effect, whereas manipulations with relatively peripheral effects should mainly influence the non-decision time T_{er}.

Method

Data sets

Tables 1 and 2 summarize the data sets to which the three different RT models were fit[3].

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[2] For the mathematical derivation of the EZ diffusion model and more details about prerequisites, assumptions, and parameter recovery, see Wagenmakers et al. (2007).

[3] Comparisons of EZ diffusion model parameters were not possible for the experiments from Miller and Ulrich (1998), Miller and Low (2001), or Miller (2012), because those experiments used simple, go/no-go, 4-choice, and 6-choice tasks, whereas the EZ diffusion model is only applicable to 2-choice tasks.

These experiments were chosen from the second author's previously published results based on (a) having at least one experimental manipulation with a clearly identifiable locus of effect, and (b) having a relatively large number of participants and trials in order to be able to obtain stable parameter estimates. In the interests of conciseness, we only describe the experimental methods and results briefly here; interested readers can consult the original articles for additional details.

The experiments shown in Table 1 included comparisons of RTs for hand versus foot responses, and in each case hand responses were significantly faster than foot responses. The experimental tasks differed widely across experiments, however. For example, the experiment coded as "HF1" (i.e., Experiment 1 of Miller, 2012) used a 4-choice RT task with the stimulus digits 1–4 assigned to left and right hand and foot responses. In contrast, HF2–HF5 (from Miller, Brookie, Wales, Wallace, & Kaup, 2018) used 2-choice RT tasks in which right hand or foot responses were indicated by stimulus word versus non-word status (HF2), by classification of a stimulus verb as being associated with hand versus foot actions (HF3), by whether a stimulus word was old or new relative to a previously memorized list (HF4), or by the color of a stimulus word (HF5). As is indicated in the table, each of the experiments also included at least one other manipulation that could affect RT, and we therefore estimated model parameters separately for each combination of participant, response limb, and the other manipulation(s).

Table 2 summarizes the experiments in which LRP had previously been used to determine whether manipulations affected processes prior to the onset of motor activation (stimulus-locked LRP effect) or subsequent to this onset (response-locked LRP effect). All used simple stimuli (e.g., single letters or shapes) assigned to response fingers on the left and right hands, but they examined quite different experimental manipulations. Some experiments examined the effects of seemingly early perceptual manipulations (e.g., stimulus intensity or discriminability, Low, Miller, & Vierck, 2002; Miller, Ulrich, & Rinkenauer, 1999), and others examined presumably late motor manipulations (e.g., simple keypress response versus a series of three keypresses, Low et al., 2002; or index versus middle versus ring response fingers, Miller & Ulrich, 1998). Still others examined the effects of what would appear to be more central decision-making processes (e.g., number of S-R alternatives, Miller & Ulrich, 1998; compatibility of S-R mappings, Miller, 2017; or different task types, Miller & Low, 2001). Finally, the comparison of Parkinson's patients versus control participants (LRP12) might well reflect differences in multiple processes.

Parameter Estimation and Analysis

Maximum likelihood estimates of the parameters of the ex-Gaussian and ex-Wald models were estimated from each participant's correct RTs in a single condition by an iterative search procedure (Rosenbrock, 1960)[4], and parameters of the EZ diffusion model were computed directly from each participant's mean and variance of correct RTs and percentage of correct responses using the equations in Wagenmakers et al. (2007). For every manipulation that produced a significant main effect or interaction in the previously-published analyses, we estimated the parameters separately for every participant x condition combination (see Appendix A for further details regarding the parameter estimation process for the ex-Gaussian and ex-Wald models). After obtaining estimates of model parameters, we followed the most common approach for the ex-Gaussian distribution and applied it to the ex-Wald distribution and the EZ diffusion model as well. More specifically, the estimated parameters for each participant x condition combination were analyzed using the ANOVA procedure, including as factors all of the experimental manipulations defining the conditions.

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[4] The average numbers of trials per cell reported in Tables 1 and 2 exclude error trials, so they are slight underestimates of the numbers of trials used for parameter estimation for the EZ diffusion model.

Results

Table B1 in the Appendix gives the mean values of all parameters, averaging across participants, for all combinations of data sets and experimental comparisons, and Table 3 gives a summary of all the effects obtained in our analyses[5]. Generally, the patterns of the results for the σ parameter were very similar to those of the μ parameter, so we focus on the results concerning μ and τ for the ex-Gaussian and ex-Wald distributions, as well as on T_D and T_{er} for the EZ diffusion model[6]. We will first consider the results from the hand versus foot experiments for all three models, and then turn to the LRP experiments. Subsequently, between-model comparisons are reported.

Hand versus Foot Experiments

Figure 1 shows the effects of the hand versus foot manipulations, separately for two parameters of each model. As can be seen in the figure, within each of the three models, effector had a significant effect on both parameters for at least one experiment.

For the ex-Gaussian distribution (Figure 1A), we found a significant effect of response effector for both μ and τ in four of the five experiments. This effect was localized to a single parameter (μ) in only one experiment (HF4), but that localization should be interpreted cautiously because that experiment had the least data (i.e., 38.00 trials per cell). The effects on μ were mostly larger (range of 44–105 ms) than those on τ (range of 16–57 ms), but this could be at least partly because the estimated Gaussian means were generally larger than the exponential means. Overall, then, these results indicate that the hand versus foot manipulation generally has an effect on both the Gaussian and exponential portions of the ex-Gaussian distribution.

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[5] We also ran a parallel set of analyses including participants who were excluded from the published analyses due to issues with EEG recording, and these analyses produced very similar results.

[6] We also separately checked the results for the drift rate v and the boundary separation a and found that the effects on T_D were mostly driven by v. These detailed results can be found in the online supplement.

Consequently, based on the assumption that this manipulation would primarily or exclusively influence motor-related processes, these results do not support the idea that the parameters of the ex-Gaussian distribution are well-suited for distinguishing between motor and more central, decisional processes.

The pattern of results for the ex-Wald distribution (Figure 1B) was somewhat different from that obtained for the ex-Gaussian distribution. Specifically, we found an effect only on μ in four of the five hand versus foot experiments, with effects on both μ (54 ms) and τ (37 ms) only in experiment HF2. In all five experiments, the effects on the ex-Wald τ (range: 8–37 ms) were numerically smaller than the effects on the ex-Gaussian τ just described. The overall pattern for the ex-Wald distribution seems somewhat clearer than the one for the ex-Gaussian distribution; that is, in four out of five experiments the μ parameter corresponds uniquely to the hand versus foot manipulation that clearly influences motor processes.

Figure 1C shows the results for the T_D and T_{er} measures of the EZ diffusion model. As mentioned in the introduction, one would expect the hand versus foot manipulation to influence only the T_{er} parameter associated with non-decisional processes, and this was indeed the result in one experiment (HF4). However, in two experiments (HF3 and HF5), we found effects on T_D as well as T_{er}. In addition, the analyses of experiment HF2 yielded a significant effect only on T_D, without an effect on T_{er}. Summarizing, the effects on T_D and T_{er} were fairly balanced; that is, over four experiments, there were three significant effects on T_D and three on T_{er} — contrary to our initial expectation of an effect primarily or entirely on T_{er}.

Figure 2 shows the results for all three models for the other manipulations that were present in the hand vs. foot experiments. Interestingly, for all three models, the increased mean RT of non-words relative to words (HF2A) seemed to be localized entirely in one model parameter (i.e., μ, μ, and T_{er}, respectively), whereas the increased mean RT of incompatible relative to compatible responses (HF5A) was localized entirely in the other model parameter (i.e., τ, τ, and T_D). Although these effects are at best mildly suggestive, they raise the possibility that these two experimental manipulations have qualitatively different effects — a possibility that would be unlikely to emerge when looking only at mean RTs. On the other hand, effects on model parameters were not always so clearcut. For the ex-Gaussian distribution, there were two cases where none of the model parameters yielded significant differences even though there was a significant effect on mean RT in the original analyses (HF3A, HF4B). There were only two comparisons where the ex-Gaussian parameters indicated statistically reliable differences: non-words vs. words, as just mentioned, and compatibility (HF5A), which had significant effects on both μ and τ, though in opposite directions (negative effect on μ and a positive effect on τ). The ex-Wald distribution also had one case with no effect on the model parameters where there was a significant effect on the mean RT (HF3A). The EZ diffusion model was the only model that produced significant effects for at least one model parameter for each of the comparisons that were significant on mean RT. Unlike Heathcote et al. (1991), we found no cases in which nonsignificant effects on RT seemed to result from counteracting effects on different model parameters. However, the conclusions that can be drawn regarding these other manipulations are quite limited, and additional replications would certainly be needed to have confidence about which model parameters were affected by each of these manipulations.

LRP Experiments

Figure 3 shows the results from the LRP experiments, separately for each model, with one row for manipulations having stimulus-locked LRP effects (A, B, C), one for those having response-locked LRP effects (D, E, F), and one for those having effects on both (G, H, I). For each of the three models, one would ideally hope that manipulations influencing stimulus-locked LRP would affect one model parameter, manipulations influencing response-locked LRP would affect the other model parameter, and manipulations influencing both types of LRP would affect both model parameters. Unfortunately, the relation between LRP effects and model parameters was not this systematic for any of the three models.

As can be seen in Figures 3A, 3D, and 3G, most experimental manipulations affected the ex-Gaussian parameter μ, regardless of whether they influenced stimulus- or response-locked LRP. Thus, it seems clear that the ex-Gaussian μ is sensitive to the durations of processes both before and after the onset of motor activation. Although τ was affected by fewer experimental manipulations than μ, it was also sensitive to manipulations of both early and late processes. Thus, these two parameters of the ex-Gaussian model do not seem to correspond simply to specific central decision versus motor stages.

The pattern of results obtained for the ex-Wald model looks fairly similar to the one obtained for the ex-Gaussian model. Nearly all manipulations influenced the μ parameter of the ex-Wald model, regardless of whether they influenced stimulus- or response-locked LRP. τ was again affected by fewer experimental manipulations than μ, but it was affected by manipulations that selectively influenced both early and late processes[7].

Since the EZ diffusion model is only applicable to two-choice tasks, we were limited to just six different comparisons for the LRP experiments. Of the two manipulations influencing stimulus-locked LRP, we found significant effects on both T_D and T_{er} for one (LRP7) and effects only on T_{er} for the other (LRP6). For the manipulations influencing response-locked LRP, we found significant effects for both parameters in all three cases (LRP11, LRP13, LRP14). One would expect motor-related processes to influence mainly the residual time T_{er} and not the decision time T_D, but that was not the case here. Thus, the parameters of the EZ diffusion model

[7] We also checked whether parameters of the shifted Wald (Heathcote, 2004) – where the non-Wald parameter is a constant shift rather than an exponential – can be linked to processing stages. Though the results across the hand vs. foot experiments were rather consistent, there was no clear pattern of results for the LRP experiments. For details see the online supplement.

do not seem to correspond very well to early and late processes either – as there was no clear-cut pattern even allowing for the fact that T_{er} can reflect both perceptual and motor processes[8].

Between Model Comparisons

We were interested not only in which parameters would be influenced by manipulations with a clear locus of effect, but also in how the different models correspond with each other. Since the ex-Gaussian and the ex-Wald distributions are quite similar, one would expect similar results for their parameters. As we described above, the result patterns obtained for those two models were generally very similar. Figure 4A provides a more detailed picture of the agreement of the ex-Gaussian distribution's μ with the ex-Wald distribution's μ, whereas Figure 4B shows the agreement for the exponential portions of the distributions. These scattergrams plot the estimates of each pair against each other, with different points representing different participants and conditions across all of the experiments. As can be seen in the figures, there were high correlations for both portions of the distributions. Most notably, even though the main difference between the ex-Gaussian and ex-Wald distributions is in the non-exponential portion of the distributions, the agreement was descriptively lower for the exponential portion (r = .770) than for the Gaussian/Wald portion (r = .922). This disagreement seems to stem from very low values for the ex-Wald exponential component with higher values for the corresponding τ estimate in the ex-Gaussian distribution.

We also compared the model fits indicated by the log likelihood from the maximum likelihood estimations for the ex-Gaussian and ex-Wald distributions for each of the 1239 RT distributions obtained for each model. The ex-Wald fit had a greater log likelihood than the ex-

[8] We also fitted the EZ2 diffusion model (Grasman, Wagenmakers, & van der Maas, 2009) – an extension to the EZ diffusion model that allows for variations in starting point z – to the same datasets as the EZ diffusion model. However, the obtained pattern of results was no more systematic regarding effects on different stages than the result pattern produced by the EZ diffusion model, so we did not include it.

Gaussian fit for 673 of the 1239 RT distributions (54.32 %), but the ex-Gaussian fits had slightly greater log likelihood on average (mean difference = 0.03, *SD* = 0.76).

Since it was unclear which of the EZ diffusion model parameters corresponds with the ex-Gaussian μ or τ and with the ex-Wald μ and τ, we compared the EZ diffusion model parameters with both parameters of each of these distributions. Naturally, these comparisons could only be made for the experiments with two choice tasks. Figures 4C–4F show the scatterplots of the EZ diffusion model's decision time T_D with the ex-Gaussian and ex-Wald parameters, whereas Figures 4G–4J show parallel scatterplots for the EZ diffusion model's residual time T_{er}. We found the decision time T_D to be more strongly linked to the exponential portions of both the ex-Gaussian distribution (Figure 4E) and the ex-Wald distribution (Figure 4F) than with the Gaussian μ (Figure 4C) and the Wald μ (Figure 4D) portions. Conversely, as can be seen in Figures 4G–4H, the residual time T_{er} of the EZ diffusion model seems to be linked to the Gaussian and Wald portions of the ex-Gaussian and ex-Wald distributions. The residual time T_{er}, on the other hand, had very low agreement with the exponential portion of the two distributions (Figures 4 I-J).

Summarizing, we found rather good correspondence between the ex-Gaussian μ and the ex-Wald μ parameters as well as for the ex-Gaussian τ and the ex-Wald τ. Moreover, the EZ diffusion model's decision time T_D seems to be linked to the exponential portion of the ex-Gaussian and ex-Wald distributions, whereas its residual time T_{er} is correlated with the Gaussian and Wald portions of the ex-Gaussian and ex-Wald distributions.

Discussion

The present study aimed at linking the parameters of the ex-Gaussian distribution, the ex-Wald distribution, and the EZ diffusion model to the effects of relatively well-understood experimental manipulations — particularly those having effects on pre-motor versus motor processing stages. There are at least three plausible arguments for trying to link these model parameters to specific processing stages. First, each of the models provides a more detailed description of observed RTs than the traditionally used mean RTs, and this can be important because experimental manipulations may sometimes appear to have no effect (i.e., on mean RTs) when they in fact influence the shapes of RT distributions (Heathcote et al., 1991). Second, the models provide good empirical fits for RT distributions (Luce, 1986; Schwarz, 2001; Wagenmakers et al., 2007), which strengthens the view that their parameters correspond meaningfully to the underlying processing stages. Third, each of the models involves additive components that could plausibly reflect different components of overall RTs.

Unfortunately, our findings suggest that none of these models are suitable for distinguishing between effects on pre-motor versus motor stages, because we found no clear pattern linking experimental manipulations to model parameters for any of the three models that we examined. That is, no parameter corresponded uniquely to manipulations with effects clearly localized either before or after the onset of motor response preparation — neither for the hand vs. foot experiments nor for the experiments in which the LRP was used for effect localization. Thus, the present results suggest that the ex-Gaussian, ex-Wald, and EZ diffusion models have limited usefulness for identifying the specific processing stages influenced by individual experimental manipulations. This finding is especially interesting for the ex-Gaussian distribution, for which the differentiation between pre-motor and motor stages has previously been made (Hohle, 1965; McGill, 1963; McGill & Gibbon, 1965; Moutsopoulou & Waszak, 2012; Singh et al., 2018) and which has been widely used to distinguish between different processing stages (e.g., Balota & Spieler, 1999; Kinoshita & Hunt, 2008; Possamaï, 1991; Steinhauser & Hübner, 2009). As tempting as it might be to use these models for separately analyzing pre-motor and motor processing stages (e.g., Singh et al., 2018), the present results reinforce earlier cautions against interpreting effects of model parameters in terms of specific processing stages (e.g., Balota & Yap, 2011; Heathcote et al., 1991; Matzke & Wagenmakers, 2009). In addition, the present finding that many experimental manipulations influence both additive components of the models reinforces the concerns of Moutsopoulou and Waszak (2012) about whether these components actually reflect temporally successive stages at all.

Although the ex-Gaussian, ex-Wald, and EZ diffusion models do not seem to accommodate separate, additive processing stages, they can still be used for other purposes, because each allows for a more fine-grained description of results than is available when just looking at the mean. The μ and T_{er} parameters seem to reflect the location of the low and middle percentiles of an RT distribution, whereas τ and T_D seem to reflect the amount of skew in the upper tail. Empirically, it is useful to distinguish manipulations with systematic effects on one of these parameters but not the other as representing different types of effects on mean RT, and there have even been cases of counteracting effects on these parameters such that manipulations had no discernible effect on the overall mean RT (e.g., Heathcote et al., 1991). As discussed in the introduction, replicable differences in effects on different model parameters (e.g., ADHD and Alzheimer research) can go beyond the mean RT to provide useful hints about how underlying processes are affected without assuming additivity of the processes reflected by the model parameters.

Finally, it is noteworthy that the parameters of the EZ diffusion model were similar to the parameters of the ex-Gaussian and ex-Wald models (Figure 4). Specifically, we found the Gaussian and Wald portions to be correlated with the non-decision time T_{er} whereas the exponential portions of the ex-Gaussian and ex-Wald models were correlated with the decision time T_D. These findings replicate and extend the results of Schmiedek, Oberauer, Wilhelm, Süß, and Wittmann (2007), who found strong relations between the ex-Gaussian τ and the drift rate of the EZ diffusion model, which is a major determinant of T_D. As emphasized by Matzke and Wagenmakers (2009), however, μ is also sensitive to some parameters affecting T_D (i.e., boundary separation *a*), so it is too simplistic to regard the two ex-Gaussian parameters as corresponding directly to T_D and T_{er}. Despite the lack of direct correspondence between RT distribution parameters and processing stages, Matzke and Wagenmakers still argued that the "descriptive use of the ex-Gaussian ... distribution is perfectly legitimate and highly encouraged" (p. 812). Though the present results are consistent with the view that the models can be useful when manipulations have selective and replicable effects on specific parameters, they emphasize that researchers should be very cautious about mapping effects of those model parameters onto processing stages.

Compliance with Ethical Standards

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Conflict of Interest

The authors declare that they have no conflict of interest.

Ethical approval

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards.

Informed consent

Informed consent was obtained from all individual participants included in the study.

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Table 1

Overview of Experiments Comparing Hand versus Foot Responses.

Label	Publication — Experiment	N	Choice Task	Other Manipulations (Label)	Trials per Cell (*M*)
HF1	Miller (2012) — 1	8	4-choice digit discrimination	Response side (HF1A)	87.22
HF2	Miller et al. (2018) — 1	16	Lexical decision	Word type (non-word, hand-/foot-word) (HF2A)	135.58
HF3	Miller et al. (2018) — 3	19	Verb classification	Stimulus type (hand-/foot-word) (HF3A)	65.13
HF4	Miller et al. (2018) — 4	23	Old/new recognition memory	Stimulus type (hand-/foot-word) (HF4A) Old/new word (HF4B)	38.00
HF5	Miller et al. (2018) — 5	20	Color discrimination	Compatibility (modified Stroop) (HF5A)	109.83

Note. Each experiment's label is used for reference within this article. Trials per cell (*M*) indicates the trials per participar condition combination used for the distributional parameter estimations and excludes error trials.

Table 2

Overview of Experiments with LRP Effects for Effect Localization.

Label	Publication — Experiment	N	Comparison	LRP Effect	Trials per Cell (*M*)
LRP1	Miller & Ulrich (1998) — 1	16	2 vs. 6 choice task	SL and RL	111.88
LRP2	Miller & Ulrich (1998) — 2	15	2 vs. 4 choice task	SL	268.47
LRP3	Miller & Ulrich (1998) — 2	15	Index options (1 vs. 3)	SL- and RL	75.78
LRP4	Miller & Ulrich (1998) — 4	30	Compatibility (incompatible vs. compatible)	RL	57.29
LRP5	Miller & Ulrich (1998) — 4	30	Response finger (left/right index, middle, ring)	RL	57.29
LRP6	Miller et al. (1999) — 1	20	Stimulus discriminability (low vs. high)	SL	173.03
LRP7	Miller (2017) — 2	24	Compatibility (incompatible vs. compatible)	SL	104.06
LRP8	Miller & Low (2001)	21	Task type: go/no-go vs. simple	SL	140.97
LRP9	Miller & Low (2001)	21	Task type: choice vs. go/no-go	SL	140.97
LRP10	Low et al. (2002)	24	Stimulus discriminability (low vs. high)	SL and RL	148.63
LRP11	Low et al. (2002)	24	Response complexity (complex vs. simple)	RL	148.63
LRP12	Low et al. (2002)	24	Group: Parkinson vs. control	SL	148.63
LRP13	Low et al. (2002)	12	Parkinson: Complex vs. simple response	RL	145.94
LRP14	Low et al. (2002)	12	Control: Complex vs. simple response	RL	151.31

Note. Each comparison's label is used for reference within this article. LRP: lateralized readiness potential, SL: stimulus-locked, RL: response-locked. The minus sign denotes an effect on LRP latency that went in the direction opposite to that of the effect on RT. Trials per cell (*M*) indicates the trials per participant x condition combination used for the distributional parameter estimations and excludes error trials.

Table 3

Effects on the model parameters for the ex-Gaussian, ex-Wald, and EZ diffusion models.

Label	Condition	ex-Gaussian			ex-Wald			EZ Diffusion	
		Δμ	Δσ	Δτ	Δμ	Δσ	Δτ	ΔT_{er}	ΔT_D
Hand vs. Foot Experiments									
HF1	Foot - hand response	81*	39*	57*	118*	64*	22		
HF2	Foot - hand response	44**	15*	47**	54*	20	37*	47	44*
HF3	Foot - hand response	92***	26*	29*	114***	45**	8	96***	24**
HF4	Foot - hand response	65***	14	16	78**	17	14	73**	19
HF5	Foot - hand response	105***	23***	29*	113***	30***	22	86***	49*
Hand vs. Foot Experiments — Other Effects									
HF1A	Left - right response	43	17	5	38	15	10		
HF2A	Non-words - words[a]	138***	6	16	131**	12	33	144***	7
HF3A	Foot - hand stimulus	25	10	19	40	15	2	17	26*
HF4A	Foot - hand stimulus	-1	-6	-6	0	-5	-6	11	-18
HF4B	New - old word	28	-22	17	37*	-22	12	45**	3
HF5A	Incompatible - compatible	-12*	-7	39***	-17	-9	43***	-11	37**
LRP Experiments									
LRP1	6 - 2 tasks	152***	47**	72*	190***	80**	36*		
LRP2	4 - 2 tasks	41**	12*	51**	45**	16*	48*		
LRP3	1 - 3 index	-10	-5	57*	4	7	43		
LRP4	Incompatible - compatible	24*	17**	54***	42*	31**	35**		
LRP5	Response finger[a]	59***	31**	37**	70***	45***	40**		
LRP6	Dim - bright stimulus	20***	-3	9	21***	-2	8	27***	1
LRP7	Incompatible - compatible	39**	14*	21**	38***	15*	22*	36**	25**
LRP8	Go-nogo - simple task	37***	9***	11	37***	10***	10		
LRP9	Choice - go-nogo task	19*	8*	27**	23*	12	24*		
LRP10	Hard - easy stimulus	38***	20***	105***	65***	37***	79***	46***	99***
LRP11	Complex - simple response	140***	40**	29	150***	49***	20	91***	79***
LRP12	Parkinson - control	102	48*	9	125*	65**	-14	72	39

Table continues on next page.

Label	Condition	ex-Gaussian			ex-Wald			EZ Diffusion	
		$\Delta\mu$	$\Delta\sigma$	$\Delta\tau$	$\Delta\mu$	$\Delta\sigma$	$\Delta\tau$	ΔT_{er}	ΔT_D
LRP13	Parkinson: complex - simple	205**	71**	33	220***	85***	17	119**	119***
LRP14	Control: complex - simple	77***	10	25	80***	13	22	63***	39*

Note. Asterisks indicate significant effects: * $p < .05$, ** $p < .01$, *** $p < .001$. Effects in milliseconds. [a] standardized effect over multiple conditions by $2\sqrt{\frac{\sum_{i=1}^{z}(x_i-\bar{x})^2}{z}}$ where z equals the number of conditions.

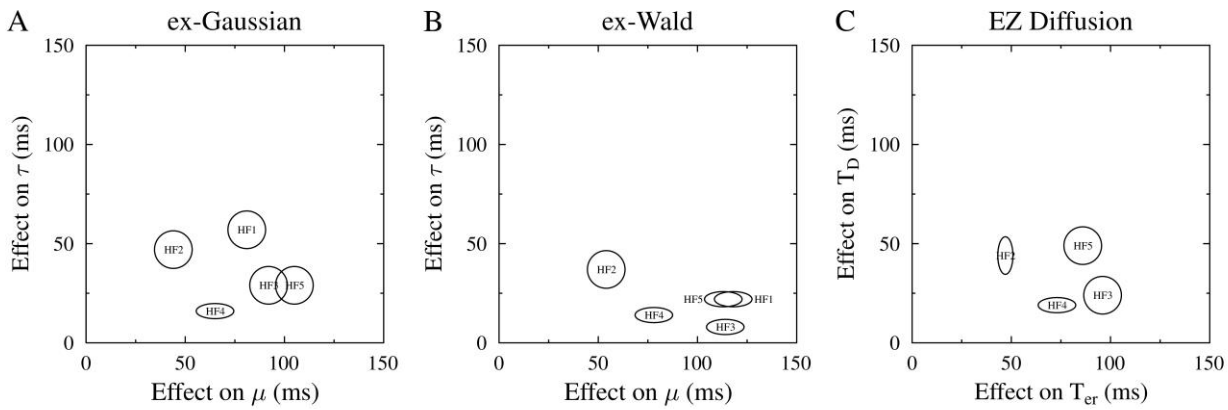

Figure 1. Effects of the hand versus foot manipulation on parameters of the ex-Gaussian (A), ex-Wald (B), and EZ diffusion (C) models. Each circle or ellipse represents one experiment. Large and small circles represent experiments in which the manipulation's effect was statistically reliable ($p < .05$) on both parameters or on neither parameter, respectively. Ellipses represent experiments in which the effect was statistically reliable only for the parameter on the axis parallel to the elongated dimension. Experiments are labeled with the codes indicated in Table 1.

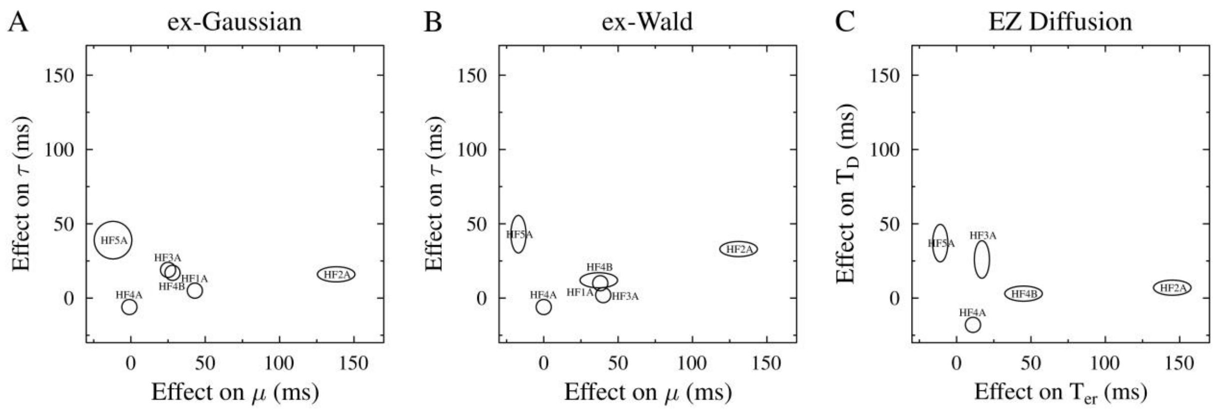

Figure 2. Effects of the other manipulations in the hand versus foot experiments on parameters of the ex-Gaussian (A), ex-Wald (B), and EZ diffusion (C) models. Each circle or ellipse represents one experiment, with size and shape reflecting statistical reliability of effects as in Figure 1. Experiments are labeled with the codes indicated in Table 1.

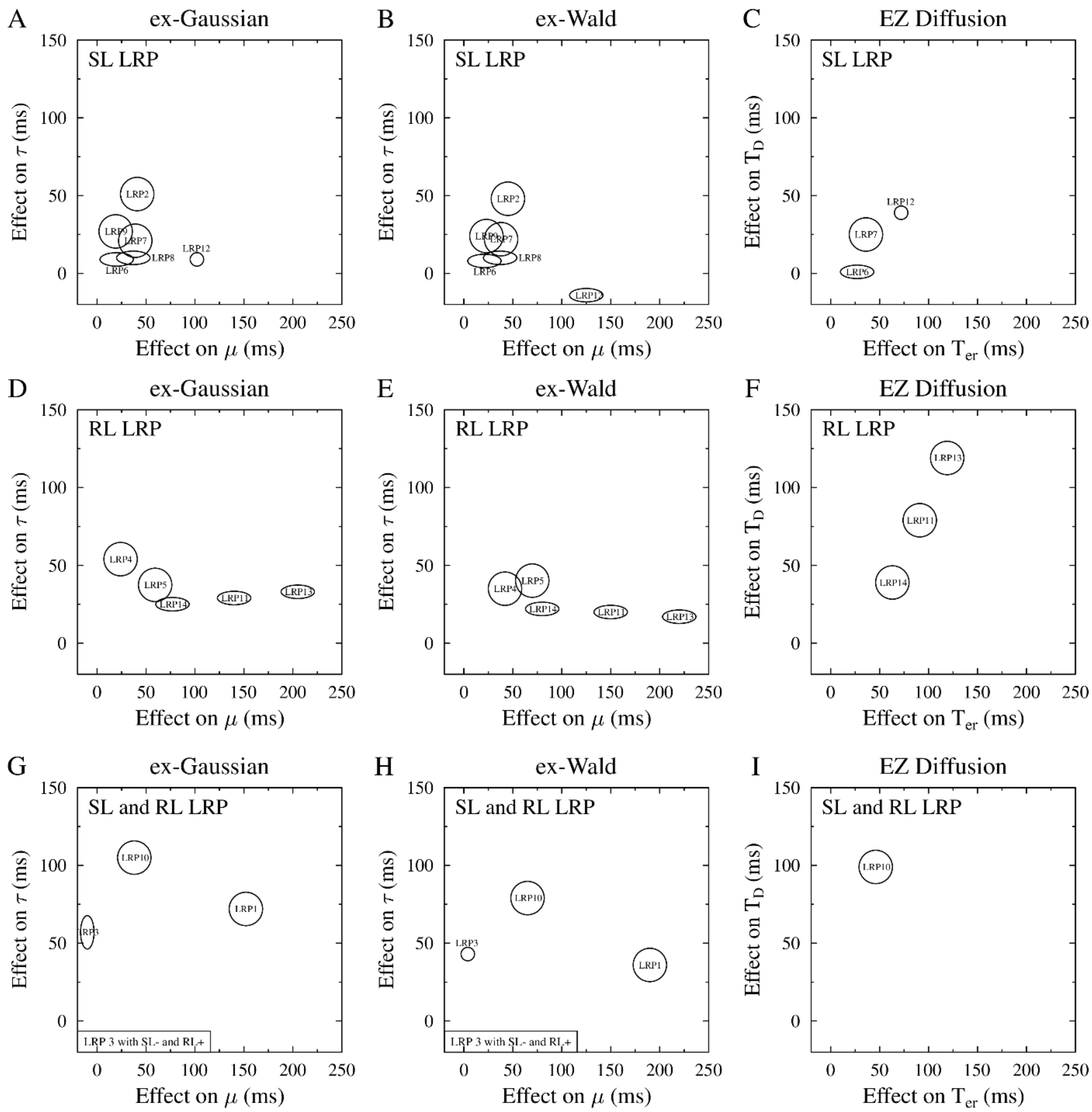

Figure 3. Effects of experimental manipulations with effects primarily on stimulus-locked lateralized readiness potentials (SL LRP; A, B, C), on response-locked lateralized readiness potentials (RL LRP; D, E, F), or both (G, H, I). Effects on parameters of the ex-Gaussian (A, D, G), ex-Wald (B, E, H), and EZ diffusion (C, F, I) models are shown separately. Each circle or ellipse represents one experiment, with size and shape reflecting statistical reliability of effects as in Figure 1. Experiments are labeled with the codes indicated in Table 2.

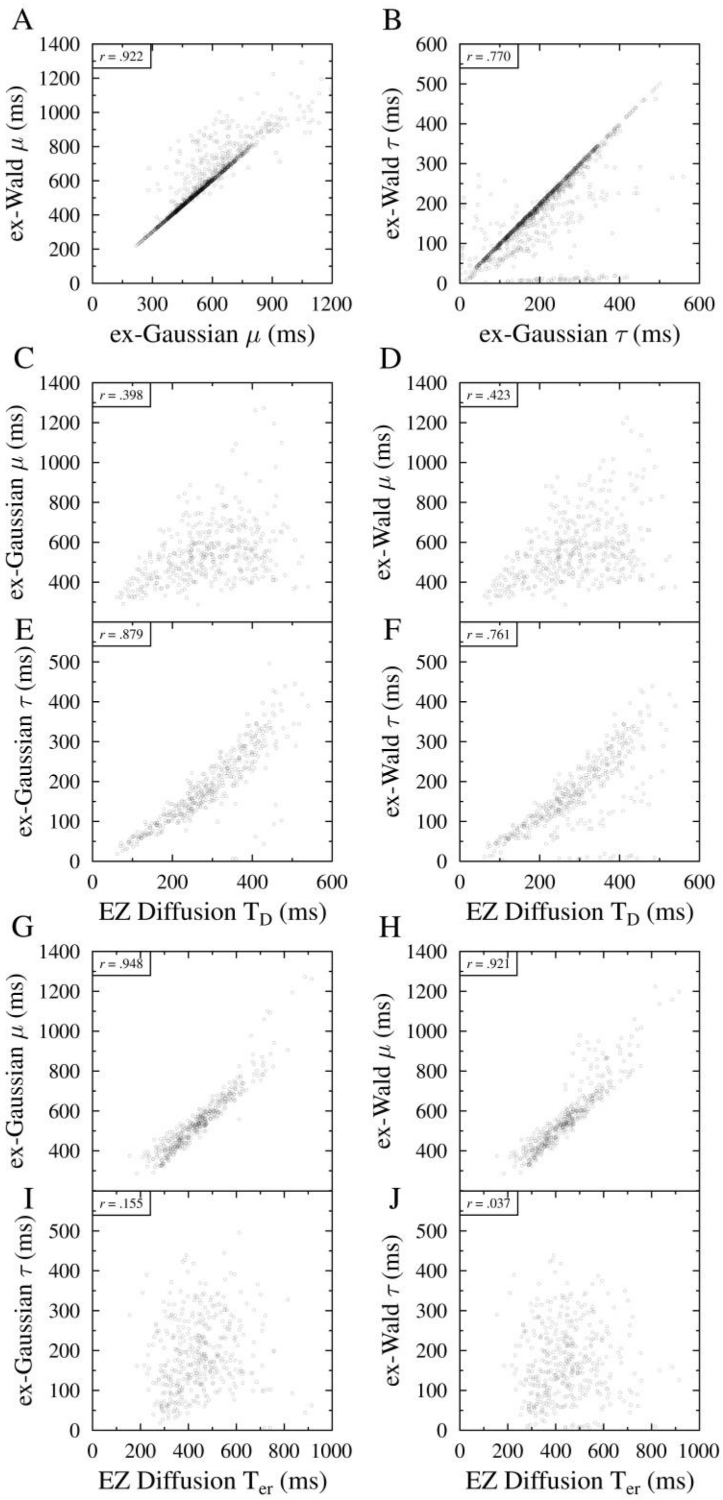

Figure 4. Scattergrams depicting the correlations between estimated parameters of different models. Each circle represents one pair of parameter estimates for a single subject x condition combination. The correlation of the Wald/Gauss portions (A) and the exponential portions (B) of the ex-Gaussian and ex-Wald distributions, the correlation of the EZ diffusion model's decision time with the Gaussian portion (C), Wald portion (D), and exponential portions (E and F) of the

ex-Gaussian and ex-Wald distributions, and the correlation of the EZ diffusion model's residual time with the Gaussian portion (G), Wald portion (H), and exponential portions (I and J) of the ex-Gaussian and ex-Wald distributions are displayed in the respective subfigures.

Appendix A

Parameter Search Process

The parameter search process was carried out using the same basic methods for both the ex-Gaussian and ex-Wald models. First, for each observed RT distribution (i.e., combination of participant and condition), individual RTs were excluded using the same criteria as in the original analyses (e.g., training blocks, minimum RT, maximum RT), and error trials were always excluded. Then, for each observed RT distribution and each model, we conducted 22 different searches for the best-fitting (i.e., maximum likelihood) parameter values, using different starting values in order to increase the probability that the search process would find the global maxima rather than ending at local maxima. We then selected the parameter values that provided the best fit across all searches for each participant x condition x model combination and used them in the further analyses.

The 22 sets of starting values for each run of the parameter search routine were chosen depending on the mean and variance of the particular set of RTs being fit. Specifically, the starting value of the exponential parameter was set to one of 22 percentages of the observed RT variance (i.e., 0.1 %, 1 %, from 10 % to 95 % in 5 % steps, 99 %, and 99.9 %). Given that value for the exponential component, the starting values of the non-exponential components were then set so that the starting ex-Gaussian or ex-Wald convolution would have mean and variance equal to the observed values for the RTs being fit. For all parameter searches, we placed a lower limit of three on the minimum value of σ for the ex-Gaussian and ex-Wald distributions, based on initial findings that the estimation process occasionally resulted in unreasonably small σ values (i.e., predicting essentially exponential RT distributions). Tarantino et al. (2013) reported a similar problem in their parameter estimations and chose to replace the unreasonable estimates with the average values for the corresponding condition.

Appendix B

Parameter Estimates per Condition

Table B1

Means of estimated model parameters by condition for the ex-Gaussian, ex-Wald, and EZ diffusion models.

Label	Condition	ex-Gaussian			ex-Wald			EZ Diffusion	
		μ	σ	τ	μ	σ	τ	T_{er}	T_D
Hand vs. Foot Experiments									
HF1	Hand response	610	76	154	637	97	126		
	Foot response	691	116	211	755	161	148		
HF2	Hand response	605	51	187	609	57	183	513	279
	Foot response	649	66	234	663	77	220	560	323
HF3	Hand response	516	65	247	544	87	218	411	352
	Foot response	608	90	276	658	132	226	507	376
HF4	Hand response	650	69	225	659	80	216	530	345
	Foot response	715	82	241	737	97	230	603	364
HF5	Hand response	404	39	156	406	43	153	306	253
	Foot response	509	63	185	519	73	175	392	302
Hand vs. Foot Experiments – Other Effects									
HF1A	Left response	672	105	185	715	136	142		
	Right response	629	88	180	677	121	132		
HF2A	Non-words	700	61	215	705	67	210	613	301
	Hand-words	555	58	198	578	74	175	456	296
	Foot-words	553	54	215	557	60	211	463	304
HF3A	Hand stimulus	549	73	252	581	102	221	451	351
	Foot stimulus	574	82	271	621	117	223	468	377

Table continues on next page.

Label	Condition	ex-Gaussian			ex-Wald			EZ Diffusion	
		μ	σ	τ	μ	σ	τ	T_{er}	T_D
HF4A	Hand stimulus	683	78	236	698	91	226	561	363
	Foot stimulus	682	73	230	698	86	220	572	345
HF4B	Old word	668	86	224	679	100	217	544	353
	New word	696	65	241	716	77	229	589	356
HF5A	Compatible	462	54	151	471	62	143	355	259
	Incompatible	450	48	190	454	53	186	344	296
LRP Experiments									
LRP1	2 choice task	447	57	157	457	65	146		
	6 choice task	599	104	229	647	145	182		
LRP2	2 choice task	427	54	137	429	59	134		
	4 choice task	468	66	188	474	75	182		
LRP3	Index only choice	461	57	226	479	76	208		
	Index 1 of 3	471	62	169	475	68	165		
LRP4	Compatible	504	85	194	534	113	164		
	Incompatible	528	102	248	576	144	199		
LRP5	Left ring finger	470	67	196	507	95	159		
	Left middle finger	495	89	227	530	122	192		
	Left index finger	564	119	210	619	164	156		
	Right index finger	513	95	235	571	143	176		
	Right middle finger	518	99	251	556	138	213		
	Right ring finger	536	92	206	549	110	193		
LRP6	Bright stimulus	375	45	79	380	50	74	304	150
	Dim stimulus	395	43	88	401	48	82	331	151
LRP7	Compatible	615	82	141	640	98	124	503	253
	Incompatible	654	96	162	678	113	146	539	278

Table continues on next page.

Label	Condition	ex-Gaussian			ex-Wald			EZ Diffusion	
		μ	σ	τ	μ	σ	τ	T_{er}	T_D
LRP8	Simple task	265	23	83	266	24	82		
LRP9	Go-nogo task	302	32	93	303	34	92		
	Choice task	321	40	120	326	46	115		
LRP10	Easy stimulus	572	74	124	576	81	119	460	235
	Hard stimulus	610	94	229	641	118	198	506	334
LRP11	Simple response	521	64	162	534	75	149	438	245
	Complex response	661	104	191	684	124	169	529	324
LRP12	Parkinson group	642	108	181	671	132	152	519	304
	Control group	540	60	172	546	67	166	447	265
LRP13	Parkinson: simple response	539	73	165	561	90	143	460	244
	Parkinson: complex response	744	144	198	781	175	160	579	363
LRP14	Control: simple response	502	55	159	506	60	155	415	246
	Control: complex response	579	65	184	586	73	177	478	285

Note. Labels refer to Tables 1 and 2.